Supplementary Appendix for "The Geography of Inventors and Local Knowledge Spillovers in R&D"

[NOT FOR PUBLICATION]

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SA1 Linked inventor biography data (INV-BIO)

The INV-BIO is comprised by approximately $150,000$ inventors in Germany with high–frequency information on their employment spells and patenting activities between 1980 and 2014. All inventors recorded in the INV-BIO data filed at least one patent with the European Patent Office (EPO) between 1999 and 2011 and were disambiguated using a combination of record linkage and machine learning methods. The INV-BIO dataset is comprised by three modules: (i) inventor module, (ii) establishment module, and (iii) patent module. I now describe details of each module.

SA1.1 Module on inventors.

The module on inventors is reported at the employment spell level. I now explain how I collapse the data at the inventor and period level. For a given inventor and year, consider the set of the inventor's spells. Then, for a given spell, the data contains information on the establishment an inventor works for, inventor's daily wage, 1-digit occupation code, whether inventor's job is part time, and the inventor's residence location. Since it is possible that an inventor reports multiple jobs within a year, an inventor's job is the one with the longest tenure. Whenever a tie happens, an inventor's job is the one with the highest daily wage. If a tie still remains, an inventor's job is chosen randomly. Part-time jobs are excluded. Finally, when collapsing the data at the inventor and period level, the last year within a period defines inventor characteristics.

SA1.2 Module on establishments.

The module on establishments is reported at the establishment and year level. I now explain how I collapse the data at the establishment and period level. The data contains a 1-digit 2008 time-consistent NACE code, the year an establishment is registered in the German administrative records for the first time, the year an establishment stops being registered in the German administrative records, and establishment location. Then, a panel of establishments is constructed based on the years the establishments were first and last registered. If the first year an establishment is registered in the data is before 1980, data on that establishment begins on 1980. If the last year an establishment is registered in the data is after 2014, data on that establishment ends on 2014. It is possible that an establishment is not registered in a given year because of lack of patenting activity by its inventors. Whenever that happens, an establishment is considered to still exists during those years, such that their industry and location are the same from the previous year. Finally, when collapsing the data at the inventor and period level, the last year within a period defines establishment characteristics.

SA1.3 Module on patents.

The module on patents is reported at the patent and inventor level. I now explain how I collapse the data at the inventor and period level. The data contains patent characteristics such as the date when the patent was filed for the first time, 2-10 forward year citations from the German Patent and Trade Mark Office (DPMA), the European Patent Office (EPO), and the United States Patent and Trademark Office (US); the mean distance between the inventors that filed the patent, 1-digit technological area, and originality and generality indices. For each patent, the earliest filing date determines the year when the patent was generated. Then, the data is collapsed at the inventor and year, such that the data reports the number of forward citations and number of filed patents during a given year. Finally, when collapsing at the inventor and period, the number of forward citations and number of filed patents are added up.

SA2 Microfoundations

In this section I provide details on the microfoundations of the model.

SA2.1 Generation of ideas

Consider an inventor i working in location o. Consider $Z_o^{i,j}$ to be the productivity of an idea j that inventor i generated. Innovation is the process where an inventor generates T_o ideas and selects the one with the highest productivity, such that

$$Z_o^i = \max_{j=1,\ldots,T_o} Z_o^{i,j}.$$

Then, the conditional probability distribution of inventor i's best idea is

$$\begin{aligned}
G(z \mid T_o) &= \Pr\left\{Z_o^{i,j} \leq z \mid T_o\right\}, \\
&= \Pr\left\{Z_o^{i,1} \leq z, \ldots, Z_o^{i\omega,T_o} \leq z \mid T_o\right\}, \\
&= \Pr\left\{Z_o^{i,1} \leq z\right\} \times \cdots \times \Pr\left\{Z_o^{i,T_o} \leq z\right\}, \\
&= \underbrace{F(z) \times \cdots \times F(z)}_{T_o \ times}, \\
&= F(z)^{T_o},
\end{aligned}$$

where $F(z)$ is the cumulative probability that an idea drawn by inventor i is below productivity z. Since T_o is the discrete number of ideas drawn by and inventor, I assume that T_o follows a Poisson distribution, such that $\Pr\{T_o = n\} = \frac{\lambda_o^n \exp(-\lambda_o)}{n!}$, where n is the number of drawn ideas, and λ_o is the expected number of drawn ideas. Additionally, I assume that ideas are drawn from a Pareto distribution, such that $F(z) = 1 - z^{-\alpha}$, where $\alpha > 1$ is a shape parameter. Then, the unconditional distribution of the productivity of inventor i's best idea is

$$\begin{aligned}
G(z) &= \Pr\left\{Z_o^i \leq z\right\}, \\
&= \sum_{n=0}^{\infty} \left[\frac{\lambda_o^n \exp(-\lambda_o)}{n!}\right] [F(z)^n], \\
&= \exp(-\lambda_o) \left[\sum_{n=0}^{\infty} \frac{(\lambda_o F(z))^n}{n!}\right], \\
&= \exp(-\lambda_o) \exp(\lambda_o F(z)), \\
&= \exp(-\lambda_o (1 - F(z))), \\
&= \exp\left(-\lambda_o \left(1 - \left(1 - z^{-\alpha}\right)\right)\right), \\
&= \exp\left(-\lambda_o z^{-\alpha}\right).
\end{aligned}$$

That is, Z_o^i is drawn from a Frechet distribution with shape parameter α and scale $\lambda_o^{\frac{1}{\alpha}}$.

SA2.2 Microfoundations: quality of intermediate inputs

Microfoundation 1: necessary tasks. A unit of an intermediate input is produced according to a blueprint. A blueprint is defined as a continuum of tasks $\mathcal{T} \equiv [0,1]$ that are necessary to produce the input at a given quality. Then, the quality of the unit of an intermediate input is

$$Z_o = \exp\left(\int_{\mathcal{T}} \log\left(Z_o^\tau\right) d\tau\right),$$

where Z_o^τ is the quality of task $\tau \in \mathcal{T}$ the intermediate input's blueprint. The firm hires a mass of inventors R_o, and each inventor generates an idea that determines the quality of each task within the blueprint. Ideas are heterogeneous in productivity and each idea improves the quality of all tasks within the blueprint, such that the quality of each task is

$$Z_o^\tau = z^\tau R_o,$$

where z^{τ} is the productivity of each idea generated by firms' inventors. Plugging this into the expression for Z_o yields

$$\begin{aligned}
Z_o &= \exp\left(\int_{\mathcal{T}} \log\left(Z_o^{\tau}\right) dt\right), \\
&= \exp\left(\int_{\mathcal{T}} \log\left(z^{\tau} R_o\right) dt\right), \\
&= \exp\left(\int_{\mathcal{T}} \left[\log\left(z^{\tau}\right) + \log\left(R_o\right)\right] dt\right), \\
&= \exp\left(\int_{\mathcal{T}} \log\left(z^{\tau}\right) d\tau + \int_{\mathcal{T}} \log\left(R_o\right) dt\right), \\
&= \exp\left(\int_{\mathcal{T}} \log\left(z^{\tau}\right) d\tau\right) \exp\left(\int_{\mathcal{T}} \log\left(R_o\right) dt\right), \\
&= \exp\left(\int_{\mathcal{T}} \log\left(z^{\tau}\right) d\tau\right) \exp\left(\log\left(R_o\right) \int_{\mathcal{T}} dt\right), \\
&= \exp\left(\int_{\mathcal{T}} \log\left(z^{\tau}\right) d\tau\right) \exp\left(\log\left(R_o\right)\right), \\
&= \exp\left(\int_{\mathcal{T}} \log\left(z^{\tau}\right) d\tau\right) R_o.
\end{aligned}$$

Since z^{τ} are draws from a Frechet distribution, then $\log\left(z^{\tau}\right)$ are draws from a Gumbel distribution with location parameter $\log\left(\lambda_o^{\frac{1}{\alpha}}\right)$ and scale parameter $\frac{1}{\alpha}$. Then,

$$\begin{aligned}
Z_o^{\omega} &= \exp\left(\int_{\mathcal{T}} \log\left(z^{\tau}\right) d\tau\right) R_o^{\omega}, \\
&= \exp\left(\int_0^{\infty} \log\left(z\right) dG_o\left(z\right)\right) R_o^{\omega}, \\
&= \exp\left(\log\left(\lambda_o^{\frac{1}{\alpha}}\right) + \frac{\overline{\gamma}}{\alpha}\right) R_o^{\omega}, \\
&= \exp\left(\log\left(\lambda_o^{\frac{1}{\alpha}}\right)\right) \exp\left(\frac{\overline{\gamma}}{\alpha}\right) R_o^{\omega}, \\
&= \psi \lambda_o^{\frac{1}{\alpha}} R_o^{\omega}.
\end{aligned}$$

where $\psi \equiv \exp\left(\frac{\overline{\gamma}}{\alpha}\right)$ is a constant, and $\overline{\gamma}$ is Euler's constant.

Microfoundation 2: linear innovation. A unit of an intermediate input is produced according to a blueprint. A blueprint is defined as a the average quality of all the ideas generated by firms' inventors. Consider that a firm hires a mass of inventors R_o. The task of each inventor is to generate ideas to be incorporated into the firm's blueprint. Inventors show up for work and form an arbitrary line, where the first inventor receives the blueprint, implements his idea into the blueprint and passes it over to the next inventor, and so on. Ideas are heterogeneous in productivity since they are drawn from a Frechet distribution. Then, the quality of intermediate input is

$$
\begin{aligned}
Z_o &= \int_o^{R_o} z^i di, \\
&= R_o \int_o^{\infty} z dG(z), \\
&= R_o \left[\Gamma\left(1 - \frac{1}{\alpha}\right) \lambda_o^{\frac{1}{\alpha}} \right], \\
&= \psi \lambda_o^{\frac{1}{\alpha}} R_o,
\end{aligned}
$$

where $\psi \equiv \Gamma\left(1 - \frac{1}{\alpha}\right)$ is a constant, and $\Gamma(\cdot)$ is the Gamma function.

SA3 Derivations

Final goods. In each location d, a representative firm produces a final good by aggregating intermediates from all locations. The production function of the final good is

$$
Q_d = \left(\sum_o Z_o^{\frac{1}{\sigma}} Q_{od}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}
$$

where Q_d is the production of the final good, Q_{od} is the quantity of intermediate inputs from o sold to the final good firm in d, Z_o is the quality of the intermediate input, and $\sigma > 1$ is the constant elasticity of substitution (CES) across intermediate inputs. The final good producer maximizes profits:

$$
\max_{\{Q_{od}\}} \; P_d Q_d - \sum_o P_{od} Q_{od} s.t.
$$
$$
Q_d = \left(\sum_o Z_o^{\frac{1}{\sigma}} Q_{od}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.
$$

The first order condition of buying an intermediate input from o is

$$
\begin{aligned}
[Q_{od}] : & P_d \left(\frac{\sigma}{\sigma-1}\right) (\ldots_d)^{\frac{\sigma}{\sigma-1}-1} Z_o^{\frac{1}{\sigma}} \left(\frac{\sigma-1}{\sigma}\right) Q_{od}^{\frac{\sigma-1}{\sigma}-1} = P_{od}, \\
P_{od} &= P_d (\ldots_d)^{\frac{1}{\sigma-1}} Z_o^{\frac{1}{\sigma}} Q_{od}^{-\frac{1}{\sigma}},
\end{aligned}
$$

where $(\ldots_d)$ is a composite of terms in d. Now, consider the first order condition of buying an intermediate input from o':

$$
P_{o'd} = P_d (\ldots_d)^{\frac{1}{\sigma-1}} Z_{o'}^{\frac{1}{\sigma}} Q_{o'd}^{-\frac{1}{\sigma}}.
$$

Divide both order conditions:

$$
\begin{aligned}
\frac{P_{od}}{P_{o'd}} &= \frac{P_d\left(\ldots_d\right)^{\frac{1}{\sigma-1}} Z_o^{\frac{1}{\sigma}} Q_{od}^{-\frac{1}{\sigma}}}{P_d\left(\ldots_d\right)^{\frac{1}{\sigma-1}} Z_{o'}^{\frac{1}{\sigma}} Q_{o'd}^{-\frac{1}{\sigma}}}, \\
\frac{P_{od}}{P_{o'd}} &= \frac{Z_o^{\frac{1}{\sigma}} Q_{od}^{-\frac{1}{\sigma}}}{Z_{o'}^{\frac{1}{\sigma}} Q_{o'd}^{-\frac{1}{\sigma}}}, \\
\frac{P_{od}}{P_{o'd}} &= \frac{Z_o^{\frac{1}{\sigma}} Q_{o'd}^{\frac{1}{\sigma}}}{Z_{o'}^{\frac{1}{\sigma}} Q_{od}^{\frac{1}{\sigma}}}, \\
\frac{P_{od}^{\sigma-1}}{P_{o'd}^{\sigma-1}} &= \frac{Z_o^{\frac{\sigma-1}{\sigma}} Q_{o'd}^{\frac{\sigma-1}{\sigma}}}{Z_{o'}^{\frac{\sigma-1}{\sigma}} Q_{od}^{\frac{\sigma-1}{\sigma}}}, \\
Q_{o'd}^{\frac{\sigma-1}{\sigma}} &= \frac{Z_{o'}^{\frac{\sigma-1}{\sigma}} Q_{od}^{\frac{\sigma-1}{\sigma}}}{Z_o^{\frac{\sigma-1}{\sigma}}} \frac{P_{od}^{\sigma-1}}{P_{o'd}^{\sigma-1}}.
\end{aligned}
$$

Plug this expression in the production function of the final good producer:

$$
\begin{aligned}
Q_d &= \left(\sum_{o'} Z_o^{\frac{1}{\sigma}} Q_{od}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \\
&= \left(\sum_{o'} Z_o^{\frac{1}{\sigma}} \frac{Z_{o'}^{\frac{\sigma-1}{\sigma}} Q_{od}^{\frac{\sigma-1}{\sigma}}}{Z_o^{\frac{\sigma-1}{\sigma}}} \frac{P_{od}^{\sigma-1}}{P_{o'd}^{\sigma-1}}\right)^{\frac{\sigma}{\sigma-1}}, \\
&= \left(\frac{Q_{od}^{\frac{\sigma-1}{\sigma}}}{Z_o^{\frac{\sigma-1}{\sigma}}} P_{od}^{\sigma-1} \sum_{o'} Z_{o'}^{\frac{1}{\sigma}} Z_{o'}^{\frac{\sigma-1}{\sigma}} P_{o'd}^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}, \\
&= \left(\frac{Q_{od}^{\frac{\sigma-1}{\sigma}}}{Z_o^{\frac{\sigma-1}{\sigma}}} P_{od}^{\sigma-1} \sum_{o'} Z_{o'} P_{o'd}^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}, \\
Q_d &= \left(\frac{Q_{od}^{\frac{\sigma-1}{\sigma}}}{Z_o^{\frac{\sigma-1}{\sigma}}} P_{od}^{\sigma-1} P_d^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}},
\end{aligned}
$$

where $P_d^{1-\sigma} = \sum_o P_{od}^{1-\sigma}$ is a CES price index. Then, rearrange this expression to obtain the demand of intermediate inputs from o:

$$
\begin{aligned}
Q_d &= \left(\frac{Q_{od}^{\frac{\sigma-1}{\sigma}}}{Z_o^{\frac{\sigma-1}{\sigma}}} P_{od}^{\sigma-1} P_d^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}, \\
&= \frac{Q_{od}}{Z_o} P_{od}^{\sigma} P_d^{-\sigma}, \\
Q_{od} &= Z_o P_{od}^{-\sigma} P_d^{\sigma} Q_d, \\
&= Z_o P_{od}^{-\sigma} P_d^{\sigma-1}\left(P_d Q_d\right), \\
Q_{od} &= Z_o P_{od}^{-\sigma} P_d^{\sigma-1} X_d,
\end{aligned}
$$

where $X_d = P_d Q_d$ is total expenditure of the final good in d.

Production of intermediate inputs. The intermediate input firm in o maximizes profits by selling its inputs to all locations subject to the demand from every location and its cost structure:

$$
\begin{aligned}
\max_{\{P_{od},Q_{od},L_{od}\}} \pi_o &= \sum_d \pi_{od}, \\
&s.t. \\
\pi_{od} &= P_{od}Q_{od} - w_o^L L_{od}, \\
L_{od} &= \frac{\tau_{od}Q_{od}}{\mathcal{A}_o}, \\
Q_{od} &= Z_o P_{od}^{-\sigma} P_d^{\sigma-1} X_d.
\end{aligned}
$$

Introduce the constraints into the profit function:

$$
\begin{aligned}
\pi_o &= \sum_d \pi_{od}, \\
&= \sum_d \left(P_{od}Q_{od} - w_o^L L_{od}\right), \\
&= \sum_d \left(P_{od}Q_{od} - \frac{\tau_{od}w_o^L}{\mathcal{A}_o} Q_{od}\right), \\
&= \sum_d \left(P_{od} Z_o P_{od}^{-\sigma} P_d^{\sigma-1} X_d\right) \\
&- \sum_d \left(\frac{\tau_{od}w_o^L}{\mathcal{A}_o} Z_o P_{od}^{-\sigma} P_d^{\sigma-1} X_d\right), \\
&= \sum_d \left(Z_o P_{od}^{1-\sigma} P_d^{\sigma-1} X_d\right) \\
&- \sum_d \left(\frac{\tau_{od}w_o^L}{\mathcal{A}_o} Z_o P_{od}^{-\sigma} P_d^{\sigma-1} X_d\right).
\end{aligned}
$$

The first order condition is

$$
\begin{aligned}
[P_{od}] : (1-\sigma)\, Z_o P_{od}^{-\sigma} P_d^{\sigma-1} X_d - (-\sigma)\, \frac{\tau_{od}w_o^L}{\mathcal{A}_o} Z_o P_{od}^{-\sigma-1} P_d^{\sigma-1} X_d &= 0, \\
0 = (1-\sigma)\, P_{od}^{-\sigma} + \sigma \frac{\tau_{od}w_o^L}{\mathcal{A}_o} P_{od}^{-\sigma-1}&, \\
(\sigma - 1) = \sigma \frac{\tau_{od}w_o^L}{\mathcal{A}_o} P_{od}^{-1}&, \\
P_{od} = \left(\frac{\sigma}{\sigma-1}\right) \frac{\tau_{od}w_o^L}{\mathcal{A}_o}&, \\
P_{od} = \overline{m} \frac{\tau_{od}w_o^L}{\mathcal{A}_o}&,
\end{aligned}
$$

where $\overline{m} \equiv \frac{\sigma}{\sigma-1}$ is the CES constant markup over marginal costs.

Firm profits. Introducing the markup pricing equation in the profit function yields

$$
\begin{aligned}
\pi_o &= \sum_d \pi_{od}, \\
&= \sum_d \left(P_{od}Q_{od} - w_o^L L_{od}\right), \\
&= \sum_d \left(P_{od}Q_{od} - \frac{\tau_{od} w_o^L}{\mathcal{A}_o} Q_{od}\right), \\
&= \sum_d \left(\overline{m}\frac{\tau_{od} w_o^L}{\mathcal{A}_o} - \frac{\tau_{od} w_o^L}{\mathcal{A}_o}\right) Q_{od}, \\
&= \sum_d \left(\overline{m} - 1\right) \frac{\tau_{od} w_o^L}{\mathcal{A}_o} Q_{od}, \\
&= \sum_d \left(\overline{m} - 1\right) \frac{\tau_{od} w_o^L}{\mathcal{A}_o} Z_o P_{od}^{-\sigma} P_d^{\sigma-1} X_d, \\
&= \sum_d \left(\overline{m} - 1\right) \frac{\tau_{od} w_o^L}{\mathcal{A}_o} Z_o \left(\overline{m}\frac{\tau_{od} w_o^L}{\mathcal{A}_o}\right)^{-\sigma} P_d^{\sigma-1} X_d, \\
&= \sum_d \left(\overline{m} - 1\right) \overline{m}^{-1} \left(\overline{m}\frac{\tau_{od} w_o^L}{\mathcal{A}_o}\right) Z_o \left(\overline{m}\frac{\tau_{od} w_o^L}{\mathcal{A}_o}\right)^{-\sigma} P_d^{\sigma-1} X_d, \\
&= Z_o \sum_d \left(\overline{m} - 1\right) \overline{m}^{-1} \left(\overline{m}\frac{\tau_{od} w_o^L}{\mathcal{A}_o}\right)^{1-\sigma} P_d^{\sigma-1} X_d, \\
&= \left(\frac{\overline{m} - 1}{\overline{m}}\right) Z_o \sum_d \left(P_{od}\right)^{1-\sigma} P_d^{\sigma-1} X_d, \\
&= \left(\frac{\frac{\sigma}{\sigma-1} - 1}{\frac{\sigma}{\sigma-1}}\right) Z_o \sum_d \left(\frac{P_{od}}{P_d}\right)^{1-\sigma} X_d, \\
&= \left(\frac{\frac{1}{\sigma-1}}{\frac{\sigma}{\sigma-1}}\right) Z_o \sum_d \left(\frac{P_{od}}{P_d}\right)^{1-\sigma} X_d, \\
&= \frac{1}{\sigma} Z_o \sum_d \left(\frac{P_{od}}{P_d}\right)^{1-\sigma} X_d, \\
&= \frac{1}{\sigma} \sum_d \left(\frac{Z_o P_{od}^{1-\sigma}}{P_d^{1-\sigma}}\right) X_d, \\
\pi_o &= \frac{1}{\sigma} \sum_d \chi_{od} X_d,
\end{aligned}
$$

where χ_{od} are the trade shares to be defined further down. Then, profits per-capita are

$$
\overline{\pi} = \frac{\sum_o \pi_o}{\sum_o \left(L_o + R_o\right)}.
$$

Research and Development (R&D). The firm R&D subsidiary maximizes research output Z_o after hiring R_o inventors at wage w_o^R, subject to its production function:

$$\begin{aligned}
&\max_{\{R_o\}} Z_o - w_o^R R_o \\
&s.t. \\
&Z_o = \psi \mathcal{Z}_o R_o^{\widetilde{\gamma}} R_o.
\end{aligned}$$

Firm's demand for inventors is

$$[R_o] : w_o^R = \psi \mathcal{Z}_o R_o^{\widetilde{\gamma}}.$$

Research and Development (R&D) with subsidies. The firm R&D subsidiary maximizes research output Z_o after hiring R_o inventors at wage $(1 - s_o)\, w_o^R$, subject to its production function:

$$\begin{aligned}
&\max_{\{R_o\}} Z_o - (1 - s_o)\, w_o^R R_o \\
&s.t. \\
&Z_o = \psi \mathcal{Z}_o R_o^{\widetilde{\gamma}} R_o.
\end{aligned}$$

Firm's demand for inventors is

$$[R_o] : w_o^R = \frac{\psi \mathcal{Z}_o R_o^{\widetilde{\gamma}}}{1 - s_o}.$$

Preferences. In each location d, agents are of two types: inventors ($n = R$), or workers ($n = L$). Each agent has preferences over the local final good (q_d^n), housing (h_d^n), and type-specific location amenities ($\mathcal{B}_d^n$):

$$\begin{aligned}
&\max_{\{q_d^n, h_d^n\}} U_d = \mathcal{B}_d^n \left(\frac{q_d^n}{\beta}\right)^{\beta} \left(\frac{h_d^n}{1 - \beta}\right)^{1-\beta} \\
&s.t. P_d q_d^n + r_d h_d^n = V_d^n,
\end{aligned}$$

where V_d^n is the agent's income, and β is the expenditure share on local final goods. This yields the following first order conditions:

$$\begin{aligned}
[q_d^n] : &\mathcal{B}_d^n \beta \frac{{q_d^n}^{\beta-1}}{\beta^{\beta}} \left(\frac{h_d^n}{1-\beta}\right)^{1-\beta} = \lambda_d P_d, \\
&\mathcal{B}_d^n \beta^{1-\beta} (1-\beta)^{\beta-1} {q_d^n}^{\beta-1} {h_d^n}^{1-\beta} = \lambda_d P_d; \\
[h_d^n] : &\mathcal{B}_d^n (1-\beta) \frac{{h_d^n}^{-\beta}}{(1-\beta)^{1-\beta}} \left(\frac{q_d^n}{\beta}\right)^{\beta} = \lambda_d r_d, \\
&\mathcal{B}_d^n \beta^{-\beta} (1-\beta)^{\beta} {q_d^n}^{\beta} {h_d^n}^{-\beta} = \lambda_d r_d;
\end{aligned}$$

where λ_d is the Lagrangian multiplier of the budget constraint. Combining both first order conditions yields

$$\begin{aligned}
\frac{\mathcal{B}_d^n \beta^{1-\beta} (1-\beta)^{\beta-1} q_d^{n\beta-1} h_d^{n1-\beta}}{\mathcal{B}_d^n \beta^{-\beta} (1-\beta)^{\beta} q_d^{n\beta} h_d^{n-\beta}} &= \frac{\lambda_d P_d}{\lambda_d r_d}, \\
\beta (1-\beta)^{-1} q_d^{n-1} h_d^n &= \frac{P_d}{r_d}, \\
\frac{\beta}{1-\beta} \frac{h_d^n}{q_d^n} &= \frac{P_d}{r_d}, \\
\frac{\beta}{1-\beta} r_d h_d^n &= P_d q_d^n.
\end{aligned}$$

If we plug this back in the budget constraint, we get the demand for housing:

$$\begin{aligned}
P_d q_d^n + r_d h_d^n &= V_d^n, \\
\frac{\beta}{1-\beta} r_d h_d^n + r_d h_d^n &= V_d^n, \\
\left(\frac{\beta}{1-\beta} - 1\right) r_d h_d^n &= V_d^n, \\
\frac{1}{1-\beta} r_d h_d^n &= V_d^n, \\
r_d h_d^n &= (1-\beta) V_d^n, \\
h_d^n &= (1-\beta) \frac{V_d^n}{r_d}.
\end{aligned}$$

Similarly, the demand for the final good is

$$q_d^n = \beta \frac{V_d^n}{P_d}.$$

If we plug the first order conditions in the utility function, we get

$$\begin{aligned}
U_d^n &= \mathcal{B}_d^n \left(\frac{q_d^n}{\beta}\right)^{\beta} \left(\frac{h_d^n}{1-\beta}\right)^{1-\beta}, \\
&= \mathcal{B}_d^n \left(\frac{\beta \frac{V_d^n}{P_d}}{\beta}\right)^{\beta} \left(\frac{(1-\beta) \frac{V_d^n}{r_d}}{1-\beta}\right)^{1-\beta}, \\
&= \mathcal{B}_d^n \left(\frac{V_d^n}{P_d}\right)^{\beta} \left(\frac{V_d^n}{r_d}\right)^{1-\beta}, \\
&= \frac{\mathcal{B}_d^n V_d^n}{P_d^{\beta} r_d^{1-\beta}}.
\end{aligned}$$

Income. Expenditure on housing in each location is redistributed as lump sum transfers to local workers. Therefore total income equals labor income plus expenditure on housing. A worker's income is

$$\begin{aligned}
V_d^L &= (1+\overline{\pi})\, w_d^L + (1-\beta)\, V_d^L, \\
V_d^L - (1-\beta)\, V_d^L &= (1+\overline{\pi})\, w_d^L, \\
\beta V_d^L &= (1+\overline{\pi})\, w_d^L, \\
V_d^L &= \frac{(1+\overline{\pi})\, w_d^L}{\beta}.
\end{aligned}$$

An inventor's income is

$$\begin{aligned}
V_d^R &= (1+\overline{\pi})\, w_d^R + (1-\beta)\, V_d^R, \\
V_d^R - (1-\beta)\, V_d^R &= (1+\overline{\pi})\, w_d^R, \\
\beta V_d^R &= (1+\overline{\pi})\, w_d^R, \\
V_d^R &= \frac{(1+\overline{\pi})\, w_d^R}{\beta}.
\end{aligned}$$

Then, total income of location d is

$$\begin{aligned}
Y_d &= V_d^L L_d + V_d^R R_d, \\
Y_d &= \frac{(1+\overline{\pi})\, w_d^L L_d}{\beta} + \frac{(1+\overline{\pi})\, w_d^R R_d}{\beta}, \\
Y_d &= \frac{(1+\overline{\pi})\left(w_d^L L_d + w_d^R R_d\right)}{\beta}.
\end{aligned}$$

Income with R&D subsidies. R&D subsidies are funded with a uniform labor tax τ. A worker's income is

$$\begin{aligned}
V_d^L &= (1-\tau+\overline{\pi})\, w_d^L + (1-\beta)\, V_d^L, \\
V_d^L - (1-\beta)\, V_d^L &= (1-\tau+\overline{\pi})\, w_d^L, \\
\beta V_d^L &= (1-\tau+\overline{\pi})\, w_d^L, \\
V_d^L &= \frac{(1-\tau+\overline{\pi})\, w_d^L}{\beta}.
\end{aligned}$$

An inventor's income is

$$\begin{aligned}
V_d^R &= (1-\tau+\overline{\pi})\, w_d^R + (1-\beta)\, V_d^R, \\
V_d^R - (1-\beta)\, V_d^R &= (1-\tau+\overline{\pi})\, w_d^R, \\
\beta V_d^R &= (1-\tau+\overline{\pi})\, w_d^R, \\
V_d^R &= \frac{(1-\tau+\overline{\pi})\, w_d^R}{\beta}.
\end{aligned}$$

Then, total income in location d is

$$
\begin{aligned}
Y_d &= V_d^L L_d + V_d^R R_d, \\
Y_d &= \frac{(1-\tau+\overline{\pi})\, w_d^L L_d}{\beta} + \frac{(1-\tau+\overline{\pi})\, w_d^R R_d}{\beta}, \\
Y_d &= \frac{(1-\tau+\overline{\pi})\left(w_d^L L_d + w_d^R R_d\right)}{\beta}.
\end{aligned}
$$

Aggregate quality. I define aggregate quality as the average quality of intermediates in a location. Since firms are symmetric and the mass of firms in each location is fixed, then a location's quality is

$$
\begin{aligned}
Z_o &= \mathbb{Z}_o R_o, \\
&= \psi \lambda_o^{\frac{1}{\alpha}} R_o, \\
&= \psi \mathcal{Z}_o R_o^{\widetilde{\gamma}} R_o, \\
&= \psi \mathcal{Z}_o R_o^{1+\widetilde{\gamma}}.
\end{aligned}
$$

Price indices. Then, the price index in d is

$$
\begin{aligned}
P_d^{1-\sigma} &= \sum_o Z_o P_{od}^{1-\sigma}, \\
P_d^{1-\sigma} &= \sum_o Z_o \left(\overline{m}\frac{\tau_{od} w_o^L}{\mathcal{A}_o}\right)^{1-\sigma}.
\end{aligned}
$$

Trade shares. Trade flows from o to d are

$$
\begin{aligned}
Q_{od} &= Z_o P_{od}^{-\sigma} P_d^{\sigma-1} X_d, \\
P_{od} Q_{od} &= Z_o P_{od}^{1-\sigma} P_d^{\sigma-1} X_d, \\
X_{od} &= Z_o P_{od}^{1-\sigma} P_d^{\sigma-1} X_d.
\end{aligned}
$$

Then, the share of intermediate inputs from o in location d's expenditure χ_{od} is

$$
\begin{aligned}
\chi_{od} &\equiv \frac{X_{od}}{X_d}, \\
&= \frac{Z_o P_{od}^{1-\sigma} P_d^{\sigma-1} X_d}{X_d}, \\
&= Z_o P_{od}^{1-\sigma} P_d^{\sigma-1}, \\
\chi_{od} &= \frac{Z_o P_{od}^{1-\sigma}}{P_d^{1-\sigma}}.
\end{aligned}
$$

Then, trade shares are

$$\begin{aligned}
\chi_{od} &= \frac{Z_o P_{od}^{1-\sigma}}{P_d^{1-\sigma}}, \\
&= \frac{Z_o P_{od}^{1-\sigma}}{\sum_o Z_o P_{od}^{1-\sigma}}, \\
&= \frac{Z_o \left(\overline{m}\frac{\tau_{od} w_o^L}{\mathcal{A}_o}\right)^{1-\sigma}}{\sum_o Z_o \left(\overline{m}\frac{\tau_{od} w_o^L}{\mathcal{A}_o}\right)^{1-\sigma}}, \\
&= \frac{Z_o \left(\frac{\tau_{od} w_o^L}{\mathcal{A}_o}\right)^{1-\sigma}}{\sum_o Z_o \left(\frac{\tau_{od} w_o^L}{\mathcal{A}_o}\right)^{1-\sigma}}, \\
\chi_{od} &= \frac{Z_o \mathcal{A}_o^{\sigma-1} \left(\tau_{od} w_o^L\right)^{1-\sigma}}{\sum_o Z_o \mathcal{A}_o^{\sigma-1} \left(\tau_{od} w_o^L\right)^{1-\sigma}}.
\end{aligned}$$

Equilibrium: trade balance. To close the model, in every location, total income equals total expenditure. Income Y_o is comprised by wages earned by workers and inventors:

$$Y_o = \frac{(1+\overline{\pi})\left(w_o^L L_o + w_o^R R_o\right)}{\beta}.$$

Expenditure X_o is comprised by purchased intermediates from every location d:

$$X_o = \sum_d \chi_{od} X_d.$$

In equilibrium, trade is balanced:

$$\begin{aligned}
Y_o &= X_o, \\
Y_o &= \sum_d \chi_{od} X_d, \\
Y_o &= \sum_d \chi_{od} Y_d, \\
\frac{(1+\overline{\pi})\left(w_o^L L_o + w_o^R R_o\right)}{\beta} &= \sum_d \chi_{od} \left(\frac{(1+\overline{\pi})\left(w_d^L L_d + w_d^R R_d\right)}{\beta}\right), \\
w_o^L L_o + w_o^R R_o &= \sum_d \chi_{od} \left(w_d^L L_d + w_d^R R_d\right).
\end{aligned}$$

Equilibrium with R&D subsidies: trade balance To close the model, in every location, total income equals total expenditure. Income Y_o is comprised by wages earned by workers and inventors:

$$Y_o = \frac{(1-\tau+\overline{\pi})\left(w_o^L L_o + w_o^R R_o\right)}{\beta}.$$

Expenditure X_o is comprised by purchased intermediates from every location d:

$$X_o = \sum_d \chi_{od} X_d.$$

In equilibrium, trade is balanced:

$$
\begin{aligned}
Y_o &= X_o, \\
Y_o &= \sum_d \chi_{od} X_d, \\
Y_o &= \sum_d \chi_{od} Y_d, \\
\frac{(1-\tau+\overline{\pi})\left(w_o^L L_o + w_o^R R_o\right)}{\beta} &= \sum_d \chi_{od} \left(\frac{(1-\tau+\overline{\pi})\left(w_d^L L_d + w_d^R R_d\right)}{\beta} \right), \\
w_o^L L_o + w_o^R R_o &= \sum_d \chi_{od} \left(w_d^L L_d + w_d^R R_d\right).
\end{aligned}
$$

Equilibrium: housing market. Aggregate demands for housing for workers and inventors in location d are $H_d^L = h_d^L L_d$ and $H_d^R = h_d^R R_d$. Then, aggregate demand for housing in location d is

$$
\begin{aligned}
H_d &= H_d^L + H_d^R, \\
H_d &= h_d^L L_d + h_d^R R_d, \\
H_d &= (1-\beta)\frac{V_d^L}{r_d} L_d + (1-\beta)\frac{V_d^R}{r_d} R_d, \\
H_d &= (1-\beta)\frac{V_d^L L_d + V_d^R R_d}{r_d}, \\
H_d &= (1-\beta)\frac{\frac{(1+\overline{\pi})w_d^L}{\beta} L_d + \frac{(1+\overline{\pi})w_d^R}{\beta} R_d}{r_d}, \\
H_d &= \left(\frac{1-\beta}{\beta}\right)\frac{(1+\overline{\pi})\, w_d^L L_d + (1+\overline{\pi})\, w_d^R R_d}{r_d}, \\
r_d &= \left(\frac{1-\beta}{\beta}\right)\frac{(1+\overline{\pi})\left(w_d^L L_d + w_d^R R_d\right)}{H_d}.
\end{aligned}
$$

Equilibrium with R&D subsidies: housing market with R&D subsidies. Aggregate demand for housing is

$$
\begin{aligned}
H_d &= H_d^L + H_d^R, \\
H_d &= h_d^L L_d + h_d^R R_d, \\
H_d &= (1-\beta)\frac{V_d^L}{r_d} L_d + (1-\beta)\frac{V_d^R}{r_d} R_d, \\
H_d &= (1-\beta)\frac{V_d^L L_d + V_d^R R_d}{r_d}, \\
H_d &= (1-\beta)\frac{\frac{(1-\tau+\overline{\pi})w_d^L}{\beta} L_d + \frac{(1-\tau+\overline{\pi})w_d^R}{\beta} R_d}{r_d}, \\
H_d &= \left(\frac{1-\beta}{\beta}\right)\frac{(1-\tau+\overline{\pi})\, w_d^L L_d + (1-\tau+\overline{\pi})\, w_d^R R_d}{r_d}, \\
r_d &= \left(\frac{1-\beta}{\beta}\right)\frac{(1-\tau+\overline{\pi})\left(w_d^L L_d + w_d^R R_d\right)}{H_d}.
\end{aligned}
$$

Equilibrium: labor market We now derive the aggregate demand for inventors. From the demand for inventors:

$$
\begin{aligned}
w_o^R &= \psi \mathcal{Z}_o R_o^{\widetilde{\gamma}}, \\
w_o^R R_o &= \psi \mathcal{Z}_o R_o^{1+\widetilde{\gamma}}, \\
w_o^R R_o &= Z_o, \\
w_o^R &= \frac{Z_o}{R_o}.
\end{aligned}
$$

Equilibrium with R&D subsidies: labor market We now derive the income for inventors. From the demand for inventors:

$$
\begin{aligned}
w_o^R &= \frac{\psi \mathcal{Z}_o R_o^{\widetilde{\gamma}}}{1-s_o}, \\
w_o^R R_o &= \frac{\psi \mathcal{Z}_o R_o^{1+\widetilde{\gamma}}}{1-s_o}, \\
w_o^R R_o &= \frac{Z_o}{1-s_o}, \\
w_o^R &= \frac{Z_o}{(1-s_o) R_o}.
\end{aligned}
$$

SA4 Optimal R&D subsidies

In this section we describe how we back out an optimal set of R&D subsidies $\{s_o^*\}$ that minimizes the distance between a competitive equilibrium with R&D subsidies and the social planner problem.

SA4.1 Social planner problem

In this section we describe the social planner problem. The planner maximizes welfare. To do this, the planner maximizes a expected utilities of workers and inventors weighted by their population, such that

$$
\max_{\{q_o^L, q_o^R, h_o^L, h_o^R, L_o, R_o, Q_{od}\}} \quad \sum_{n=\{L,R\}} \sum_o \left(\frac{n_o}{\overline{N}} \right) \overline{U}_o^n,
$$

where $n_o = \{L_o, R_o\}$ is the population of workers and inventors in location o, $\overline{N}$ is total exogenous population, and $\overline{U}_o^n$ is the expected utility of agent type n in location o. Following the properties of the Frechet distribution , expected utility is

$$
\overline{U}_o^n = \Gamma \left(\frac{\kappa - 1}{\kappa} \right) \left(\sum_d \left(\frac{U_d^n}{\mu_{od}^n} \right)^{\kappa} \right)^{\frac{1}{\kappa}},
$$

where utility is

$$
U_d^n = \mathcal{B}_d^n \left(\frac{q_d^n}{\beta} \right)^{\beta} \left(\frac{h_d^n}{1-\beta} \right)^{1-\beta}.
$$

The social planner maximizes welfare subject to a set of market clearing conditions. In particular, the labor market for workers and inventors in each location d clears such that

$$L_d = \sum_o \left(\frac{\left(\frac{U_d^L}{\mu_{od}^L}\right)^{\kappa}}{\sum_\delta \left(\frac{U_\delta^L}{\mu_{o\delta}^L}\right)^{\kappa}} \right) \overline{L}_o,$$

$$R_d = \sum_o \left(\frac{\left(\frac{U_d^R}{\mu_{od}^R}\right)^{\kappa}}{\sum_\delta \left(\frac{U_\delta^R}{\mu_{o\delta}^R}\right)^{\kappa}} \right) \overline{R}_o.$$

The housing market in each location o clears, such that

$$\overline{H}_o = h_o^L L_o + h_o^R R_o.$$

The final good market in each location d clears, such that

$$Q_d = q_d^L L_d + q_d^R R_d,$$

where

$$Q_d = \left(\sum_o Z_o^{\frac{1}{\sigma}} Q_{od}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

Finally, trade balance for intermediate inputs holds, such that

$$\mathcal{A}_o L_o = \sum_d \tau_{od} Q_{od}.$$

This maximization yields an equilibrium $\widehat{X}_o \equiv \left\{ \widehat{q_o^L}, \widehat{q_o^R}, \widehat{h_o^L}, \widehat{h_o^R}, \widehat{L_o}, \widehat{R_o}, \widehat{Q_{od}} \right\}$.

SA4.2 Optimal R&D subsidies

Consider a competitive equilibrium with R&D subsidies $X_o^* \equiv \left\{ q_o^{L^*}, q_o^{R^*}, h_o^{L^*}, h_o^{R^*}, L_o^*, R_o^*, Q_{od}^* \right\}$ at a given R&D subsidy s_o. Then, the optimal R&D subsidies s_o^* are such that

$$s_o^* = \arg\min_{\{s_o\}} \sum_o \left(X_o^* - \widehat{X}_o \right)^2.$$

SA5 Taking the Model to the Data

Local Knowledge Spillovers in R&D $\{\widetilde{\gamma}\}$. Inventor productivity can be written as

$$
\begin{aligned}
\mathbb{E}\left\{Z_o^i\right\} &= \psi\lambda_o^{\frac{1}{\alpha}}, \\
&= \psi\mathcal{Z}_o R_o^{\widetilde{\gamma}}, \\
Z_o^i &= \psi\mathcal{Z}_o R_o^{\widetilde{\gamma}} \exp\left(\epsilon_o^i\right), \\
\log\left(Z_o^i\right) &= \log\left(\psi\mathcal{Z}_o R_o^{\widetilde{\gamma}} \exp\left(\epsilon_o^i\right)\right), \\
&= \log(\psi) + \log(\mathcal{Z}_o) + \log\left(R_o^{\widetilde{\gamma}}\right) + \log\left(\exp\left(\epsilon_o^i\right)\right), \\
&= \underbrace{\log(\psi)}_{\equiv\iota} + \underbrace{\log(\mathcal{Z}_o)}_{\equiv\iota_o} + \widetilde{\gamma}\log(R_o) + \epsilon_o^i, \\
&= \iota + \iota_o + \widetilde{\gamma}\log(R_o) + \epsilon_o^i.
\end{aligned}
$$

After considering the additional time dimension t and technological areas a, and first differences, this expression is the model counterpart of the empirical specification used to estimate local knowledge spillovers in R&D ($\beta = 0.409$).

Migration costs $\left\{\mu_{od}^R, \mu_{od}^L\right\}$. For inventors ($n = R$) and workers ($n = L$), I parametrize migration costs as an exponential function of geographic distance between every location pair $\mu_{od}^n = \rho_0^n dist_{od}^{\rho_1^n} \exp\left(-\frac{\epsilon_{od}^n}{\kappa}\right)$, where $\{\rho_0^n\}$ are intercepts that determines the overall level of internal migration, $\{\rho_1^n\}$ are the elasticities of migration costs to distance, and ϵ_{od}^n are i.i.d. shocks. The intercepts are calibrated by targeting the overall migration rate for workers and inventors. To estimate the migration cost elasticities, consider migration shares such that

$$
\begin{aligned}
\eta_{od}^{n} &= \frac{\left(\frac{U_d^n}{\mu_{od}^n}\right)^{\kappa}}{\sum_{\delta}\left(\frac{U_{\delta}^n}{\mu_{o\delta}^R}\right)^{\kappa}}, \\
\log\left(\eta_{od}^{n}\right) &= \log\left(\frac{\left(\frac{U_d^n}{\mu_{od}^n}\right)^{\kappa}}{\sum_{\delta}\left(\frac{U_{\delta,t}^n}{\mu_{o\delta}^n}\right)^{\kappa}}\right), \\
&= \log\left(\left(\frac{U_d^n}{\mu_{od}^n}\right)^{\kappa}\right) - \log\left(\sum_{\delta}\left(\frac{U_{\delta}^n}{\mu_{o\delta}^n}\right)^{\kappa}\right), \\
&= \kappa\log\left(\frac{U_d^n}{\mu_{od}^n}\right) - \log\left(\sum_{\delta}\left(\frac{U_{\delta}^n}{\mu_{o\delta}^n}\right)^{\kappa}\right), \\
&= -\kappa\log\left(\mu_{od}^n\right) \underbrace{-\log\left(\sum_{\delta}\left(\frac{U_{\delta}^R}{\mu_{o\delta}^n}\right)^{\kappa}\right)}_{=\iota_o} + \underbrace{\kappa\log\left(U_d^n\right)}_{=\iota_d}, \\
&= \iota_o + \iota_d - \kappa\log\left(\mu_{od}^n\right), \\
&= \iota_o + \iota_d - \kappa\log\left(\rho_0^n dist_{od}^{\rho_1^n}\exp\left(-\frac{\epsilon_{od}^n}{\kappa}\right)\right), \\
&= \iota_o + \iota_d \underbrace{-\kappa\log\left(\rho_0^n\right)}_{\iota} - \kappa\log\left(dist_{od}^{\rho_1^n}\right) - \kappa\log\left(\exp\left(-\frac{\epsilon_{od}^n}{\kappa}\right)\right), \\
&= \iota + \iota_o + \iota_d - \kappa\rho_1^n\log\left(dist_{od}\right) + \epsilon_{od}^n.
\end{aligned}
$$

This migration gravity equation states that, conditional on data on migration shares $\{\eta_{od}^n\}$, geographic distances $\{dist_{od}\}$, the migration elasticity $\{\kappa\}$, and the inclusion of origin and destination fixed effects $\{\iota_o, \iota_d\}$, then migration cost elasticities $\{\rho_1^n\}$ are identified.

Location fundamentals $\{\mathcal{Z}_o, \mathcal{A}_o\}$. I recover location fundamentals for R&D $\{\mathcal{Z}_o\}$ and production $\{\mathcal{A}_o\}$ through model inversion. Given parameter values $\{\psi, \widetilde{\gamma}\}$, and data on wages and population $\left\{w_o^R, R_o\right\}$, there is a unique set of values for location fundamentals for R&D $\{\mathcal{Z}_o\}$ that that is consistent with the aggregate demand for inventors. Then, given trade costs $\{\tau_{od}\}$, location fundamentals for R&D $\{\mathcal{Z}_o\}$, parameter values $\{\psi, \sigma, \widetilde{\gamma}\}$, and data on wages and population $\left\{w_o^R, w_o^L, R_o, L_o\right\}$, there is a unique set of values for location fundamental for production $\{\mathcal{A}_o\}$ that is consistent with the data. Given equilibrium in goods market, trade shares, and aggregate productivity, I construct the following system of excess demand functions:

$$
\begin{aligned}
\mathbb{D}_o\left(\mathcal{A}\right) &\equiv w_o^L L_o + w_o^R R_o - \sum_d \chi_{od}\left(w_d^L L_d + w_d^R R_d\right), \\
&= w_o^L L_o + w_o^R R_o - \sum_d \left(\frac{Z_o \mathcal{A}_o^{\sigma-1}\left(\tau_{od} w_o^L\right)^{1-\sigma}}{\sum_o Z_o \mathcal{A}_o^{\sigma-1}\left(\tau_{od} w_o^L\right)^{1-\sigma}}\right)\left(w_d^L L_d + w_d^R R_d\right),
\end{aligned}
$$

where $Z_o = \psi\mathcal{Z}_o R_o^{1+\widetilde{\gamma}}$. It can be shown that this excess demand functions are (i) continuous, (ii) homogeneous of degree zero, (iii) $\sum_o \mathbb{D}_o\left(\mathcal{A}\right) = 0$, and (iv) $\frac{\partial\mathbb{D}_o(\mathcal{A})}{\partial\mathcal{A}_l} > 0, \forall o, l \in \mathcal{S}, \mathcal{S}, l \neq o$ and $\frac{\partial\mathbb{D}_o(\mathcal{A})}{\partial\mathcal{A}_o} < 0, \forall o \in \mathcal{S}$. Given this properties, up to a normalization, there exists a unique vector $\mathcal{A}^*$ such that $\mathbb{D}_o\left(\mathcal{A}^*\right) = 0, \forall o \in \mathcal{S}$. I use data on wages and population $\left\{w_o^R, w_o^L, R_o, L_o\right\}$ for year 2014.

Location amenities $\left\{\mathcal{B}_o^R, \mathcal{B}_o^L\right\}$**.** I recover location amenities for both workers and inventors $\left\{\mathcal{B}_o^R, \mathcal{B}_o^L\right\}$ through model inversion. Given the exogenous distribution of workers and inventors across locations $\left\{\overline{R}_o, \overline{L}_o\right\}_{\forall o \in \mathcal{S}}$, fixed supply of housing $\left\{\overline{H}_o\right\}_{\forall o \in \mathcal{S}}$, trade costs $\{\tau_{od}\}$, migration costs $\left\{\mu_{od}^R, \mu_{od}^L\right\}$, fundamental location productivities $\{\mathcal{Z}_o, \mathcal{A}_o\}$, parameter values $\{\alpha, \psi, \kappa, \sigma, \beta\}$, and data on wages and population $\left\{w_o^R, w_o^L, R_o, L_o\right\}$, there is a unique set of values for fundamental location amenities $\left\{\mathcal{B}_o^R, \mathcal{B}_o^L\right\}$ that is consistent with the data. The initial distribution $\left\{\overline{R}_o, \overline{L}_o\right\}$ is from 1980 and they are scaled such that the total number of workers and inventors in West Germany is the same for 2014. To simplify, I consider that $\overline{H}_o = \overline{L}_o + \overline{R}_o$. Given labor supply functions, migration shares, and indirect utility functions, I construct the following system of excess demand functions:

$$
\begin{aligned}
\mathbb{D}_d^R\left(\mathcal{B}^R\right) &= R_d - \sum_o \eta_{od}^R \overline{R}_o, \\
&= R_d - \sum_o \left(\frac{\left(\frac{U_d^R}{\mu_{od}^R}\right)^{\kappa}}{\sum_\delta \left(\frac{U_\delta^R}{\mu_{o\delta}^R}\right)^{\kappa}} \right) \overline{R}_o, \\
&= R_d - \sum_o \left(\frac{\left(\frac{\mathcal{B}_d^R V_d^R}{\mu_{od}^R P_d^\beta r_d^{1-\beta}}\right)^{\kappa}}{\sum_\delta \left(\frac{\mathcal{B}_\delta^R V_\delta^R}{\mu_{o\delta}^R P_\delta^\beta r_\delta^{1-\beta}}\right)^{\kappa}} \right) \overline{R}_o, \\
&= R_d - \sum_o \left(\frac{\left(\frac{\mathcal{B}_d^R \frac{1+\overline{\pi}}{\beta} w_d^R}{\mu_{od}^R P_d^\beta r_d^{1-\beta}}\right)^{\kappa}}{\sum_\delta \left(\frac{\mathcal{B}_\delta^R \frac{1+\overline{\pi}}{\beta} w_\delta^R}{\mu_{o\delta}^R P_\delta^\beta r_\delta^{1-\beta}}\right)^{\kappa}} \right) \overline{R}_o, \\
&= R_d - \sum_o \left(\frac{\left(\frac{\mathcal{B}_d^R w_d^R}{\mu_{od}^R P_d^\beta r_d^{1-\beta}}\right)^{\kappa}}{\sum_\delta \left(\frac{\mathcal{B}_\delta^R w_\delta^R}{\mu_{o\delta}^R P_\delta^\beta r_\delta^{1-\beta}}\right)^{\kappa}} \right) \overline{R}_o.
\end{aligned}
$$

The same procedure can be applied for workers:

$$
\mathbb{D}_d^L\left(\mathcal{B}^L\right) = L_d - \sum_o \left(\frac{\left(\frac{\mathcal{B}_d^L w_d^L}{\mu_{od}^L P_d^\beta r_d^{1-\beta}}\right)^{\kappa}}{\sum_\delta \left(\frac{\mathcal{B}_\delta^L w_\delta^L}{\mu_{o\delta}^L P_\delta^\beta r_\delta^{1-\beta}}\right)^{\kappa}} \right) \overline{L}_o,
$$

where $P_d^{1-\sigma} = \overline{m} \sum_o Z_o \mathcal{A}_o^{\sigma-1} \left(\tau_{od} w_o^L\right)^{1-\sigma}$ and $r_d = \left(\frac{1-\beta}{\beta}\right) \frac{(1+\overline{\pi})\left(w_d^L L_d + w_d^R R_d\right)}{\overline{H}_d}$. It can be shown that these excess demand functions are (i) continuous, (ii) homogeneous of degree zero, (iii) $\sum_o \mathbb{D}_d^n\left(\mathcal{B}^n\right) = 0$, and (iv) $\frac{\partial \mathbb{D}_d^n\left(\mathcal{B}^n\right)}{\partial \mathcal{B}_l^n} > 0, \forall d, l \in \mathcal{S}, \mathcal{S}, l \neq o$ and $\frac{\partial \mathbb{D}_d^n\left(\mathcal{B}^n\right)}{\partial \mathcal{B}_d^n} < 0, \forall d \in \mathcal{S}$. Given this properties, up to a normalization, there exists a unique vector $\mathcal{B}^{n^*}$ such that $\mathbb{D}_d^n\left(\mathcal{B}^{n^*}\right) = 0, \forall d \in \mathcal{S}, n = \{L, R\}$. I use data on wages and population $\left\{w_o^R, w_o^L, R_o, L_o\right\}$ for year 2014.

SA6 Solution algorithms

In this section I describe the algorithms that solve the model. The supra-script (i) denotes a variable as an "input", and the supra-script (o) denotes a variable as an "output".

SA6.1 Equilibrium

Given the exogenous distribution of workers and inventors across locations $\{\overline{L}_o, \overline{R}_o\}_{\forall o\in\mathcal{S}}$, location fundamentals $\{\mathcal{Z}_o, \mathcal{A}_o\}_{\forall o\in\mathcal{S}}$, location amenities $\{\mathcal{B}^L_o, \mathcal{B}^R_o\}_{\forall o\in\mathcal{S}}$, migration costs $\{\mu^n_{od}\}^{n=\{L,R\}}_{\forall o,d\in\mathcal{S},\mathcal{S}}$, trade costs $\{\tau_{od}\}_{\forall o,d\in\mathcal{S},\mathcal{S}}$, and parameters, guess $\left\{w^{L^{(i)}}_o, w^{R^{(i)}}_o, Z^{(i)}_o, r^{(i)}_o\right\}_{\forall o\in\mathcal{S}}$ and $\left\{\overline{\pi}^{(i)}\right\}$ and follow these steps:

1. Bilateral price indices $\{P_{od}\}_{\forall o,d\in\mathcal{S},\mathcal{S}}$:

$$P^{1-\sigma}_{od} = \left(\overline{m}\frac{\tau_{od} w^{L^{(i)}}_o}{\mathcal{A}_o}\right)^{1-\sigma}.$$

2. New price indices $\left\{P^{(o)}_d\right\}_{\forall d\in\mathcal{S}}$:

$$P_d = \left(\sum_o Z^{(i)}_o P^{1-\sigma}_{od}\right)^{\frac{1}{1-\sigma}}.$$

3. Migration shares $\{\eta^n_{od}\}^{n=\{L,R\}}_{\forall o,d\in\mathcal{S},\mathcal{S}}$:

$$\eta^n_{od} = \frac{\left(\frac{\mathcal{B}^n_d w^{n^{(i)}}_d}{\mu^n_{od} P^\beta_d {r^{(i)}_d}^{1-\beta}}\right)^\kappa}{\sum_\delta \left(\frac{\mathcal{B}^n_\delta w^{n^{(i)}}_\delta}{\mu^n_{o\delta} P^\beta_\delta {r^{(i)}_\delta}^{1-\beta}}\right)^\kappa}.$$

4. Number of workers and inventors $\{L_d, R_d\}_{\forall d\in\mathcal{S}}$:

$$L_d = \sum_o \eta^L_{od}\overline{L}_o,$$
$$R_d = \sum_o \eta^R_{od}\overline{R}_o.$$

5. New aggregate productivity $\{Z_o\}_{\forall o\in\mathcal{S}}$ from its definition:

$$Z^{(o)}_o = \psi\mathcal{Z}_o R^{1+\widetilde{\gamma}}_o.$$

6. New housing rent $\left\{r^{(o)}_o\right\}_{\forall o\in\mathcal{S}}$ from housing equilibrium:

$$r^{(o)}_o = \left(\frac{1-\beta}{\beta}\right)\frac{\left(1+\overline{\pi}^{(i)}\right)\left(w^{L^{(i)}}_o L_o + w^{R^{(i)}}_o R_o\right)}{\overline{H}_o}.$$

7. Trade shares $\{\chi_{od}\}^{n=\{L,R\}}_{\forall o,d\in\mathcal{S},\mathcal{S}}$:

$$\chi_{od} = \frac{Z^{(i)}_o P^{1-\sigma}_{od}}{P^{1-\sigma}_d}.$$

8. New inventor wages $\left\{w^{R^{(o)}}_o\right\}_{\forall o\in\mathcal{S}}$ from inventors' income:

$$w^{R^{(o)}}_o = \frac{Z^{(i)}_o}{R_o}$$

9. New worker wages $\left\{w_o^{L^{(o)}}\right\}_{\forall o\in\mathcal{S}}$ from trade balance:

$$w_o^{L^{(o)}} L_o + w_o^{R^{(i)}} R_o = \sum_d \chi_{od} \left(w_d^{L^{(o)}} L_d + w_d^{R^{(i)}} R_d\right).$$

10. Normalize wages such that $w_1^{L^{(o)}} = 1$.

11. New profits per-capita $\left\{\overline{\pi}^{(o)}\right\}$:

$$\overline{\pi}^{(o)} = \frac{\sum_o \pi_o}{\sum_o (L_o + R_o)},$$

where

$$\pi_o = \frac{1}{\sigma} \sum_d \chi_{od} \left(w_d^{L^{(i)}} L_d + w_d^{R^{(i)}} R_d\right).$$

12. Update:

$$\begin{aligned}
w_o^{L^{(i)}} &= (0.2)\, w_o^{L^{(o)}} + (0.8)\, w_o^{L^{(i)}}, \\
w_o^{R^{(i)}} &= (0.2)\, w_o^{R^{(o)}} + (0.8)\, w_o^{R^{(i)}}, \\
Z_o^{(i)} &= (0.5)\, Z_o^{(o)} + (0.5)\, Z_o^{(i)}, \\
r_o^{(i)} &= (0.5)\, r_o^{(o)} + (0.5)\, r_o^{(i)}, \\
\overline{\pi}^{(i)} &= (0.5)\, \overline{\pi}^{(o)} + (0.5)\, \overline{\pi}^{(i)}.
\end{aligned}$$

13. Iterate until convergence is achieved.

SA6.2 Equilibrium with R&D subsidies

Given the exogenous distribution of workers and inventors across locations $\left\{\overline{L}_o, \overline{R}_o\right\}_{\forall o\in\mathcal{S}}$, location fundamentals $\{\mathcal{Z}_o, \mathcal{A}_o\}_{\forall o\in\mathcal{S}}$, location amenities $\left\{\mathcal{B}_o^L, \mathcal{B}_o^R\right\}_{\forall o\in\mathcal{S}}$, migration costs $\{\mu_{od}^n\}_{\forall o,d\in\mathcal{S},\mathcal{S}}^{n=\{L,R\}}$, trade costs $\{\tau_{od}\}_{\forall o,d\in\mathcal{S},\mathcal{S}}$, R&D subsidies $\{s_o\}_{\forall o\in\mathcal{S}}$, and parameters, guess $\left\{w_o^{L^{(i)}}, w_o^{R^{(i)}}, Z_o^{(i)}, r_o^{(i)}\right\}_{\forall o\in\mathcal{S}}$ and $\left\{\overline{\pi}^{(i)}, \tau^{(i)}\right\}$ and follows these steps:

1. Bilateral price indices $\{P_{od}\}_{\forall o,d\in\mathcal{S},\mathcal{S}}$:

$$P_{od}^{1-\sigma} = \left(\overline{m}\frac{\tau_{od} w_o^{L^{(i)}}}{\mathcal{A}_o}\right)^{1-\sigma}.$$

2. New price indices $\left\{P_d^{(o)}\right\}_{\forall d\in\mathcal{S}}$:

$$P_d = \left(\sum_o Z_o^{(i)} P_{od}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

3. Migration shares $\{\eta_{od}^n\}_{\forall o,d\in\mathcal{S},\mathcal{S}}^{n=\{L,R\}}$:

$$\eta_{od}^n = \frac{\left(\frac{\mathcal{B}_d^n w_d^{n^{(i)}}}{\mu_{od}^n P_d^\beta r_d^{(i)1-\beta}}\right)^\kappa}{\sum_\delta \left(\frac{\mathcal{B}_\delta^n w_\delta^{n^{(i)}}}{\mu_{o\delta}^n P_\delta^\beta r_\delta^{(i)1-\beta}}\right)^\kappa}.$$

4. Number of workers and inventors $\{L_d, R_d\}_{\forall d \in \mathcal{S}}$:

$$L_d = \sum_o \eta^L_{od} \overline{L}_o,$$
$$R_d = \sum_o \eta^R_{od} \overline{R}_o.$$

5. New aggregate productivity $\{Z_o\}_{\forall o \in \mathcal{S}}$ from its definition:

$$Z_o^{(o)} = \psi \mathcal{Z}_o R_o^{1+\widetilde{\gamma}}.$$

6. New housing rent $\left\{r_o^{(o)}\right\}_{\forall o \in \mathcal{S}}$ from housing equilibrium:

$$r_o^{(o)} = \left(\frac{1-\beta}{\beta}\right) \frac{\left(1 + \overline{\pi}^{(i)} - \tau^{(i)}\right)\left(w_o^{L^{(i)}} L_o + w_o^{R^{(i)}} R_o\right)}{\overline{H}_o}.$$

7. Trade shares $\{\chi_{od}\}^{n=\{L,R\}}_{\forall o,d \in \mathcal{S},\mathcal{S}}$:

$$\chi_{od} = \frac{Z_o^{(i)} P_{od}^{1-\sigma}}{P_d^{1-\sigma}}.$$

8. New inventor wages $\left\{w_o^{R^{(o)}}\right\}_{\forall o \in \mathcal{S}}$ from inventors' income:

$$w_o^{R^{(o)}} = \frac{Z_o^{(i)}}{(1 - s_o) R_o}$$

9. New worker wages $\left\{w_o^{L^{(o)}}\right\}_{\forall o \in \mathcal{S}}$ from trade balance:

$$w_o^{L^{(o)}} L_o + w_o^{R^{(i)}} R_o = \sum_d \chi_{od} \left(w_d^{L^{(o)}} L_d + w_d^{R^{(i)}} R_d\right).$$

10. Normalize wages such that $w_1^{L^{(o)}} = 1$.

11. New profits per-capita $\left\{\overline{\pi}^{(o)}\right\}$:

$$\overline{\pi}^{(o)} = \frac{\sum_o \pi_o}{\sum_o (L_o + R_o)},$$

where

$$\pi_o = \frac{1}{\sigma} \sum_d \chi_{od} \left(w_d^{L^{(i)}} L_d + w_d^{R^{(i)}} R_d\right).$$

12. New labor tax from government's balanced budget:

$$\tau^{(o)} = \frac{\sum_o s_o \left(w_o^{R^{(i)}} R_o\right)}{\sum_o \left(w_o^{L^{(i)}} L_o + w_o^{R^{(i)}} R_o\right)}.$$

13. Update:

$$
\begin{aligned}
w_o^{L^{(i)}} &= (0.2)\, w_o^{L^{(o)}} + (0.8)\, w_o^{L^{(i)}}, \\
w_o^{R^{(i)}} &= (0.2)\, w_o^{R^{(o)}} + (0.8)\, w_o^{R^{(i)}}, \\
Z_o^{(i)} &= (0.5)\, Z_o^{(o)} + (0.5)\, Z_o^{(i)}, \\
r_o^{(i)} &= (0.5)\, r_o^{(o)} + (0.5)\, r_o^{(i)}, \\
\overline{\pi}^{(i)} &= (0.5)\, \overline{\pi}^{(o)} + (0.5)\, \overline{\pi}^{(i)}, \\
\tau^{(i)} &= (0.5)\, \tau^{(o)} + (0.5)\, \tau^{(i)}.
\end{aligned}
$$

14. Iterate until convergence is achieved.